Name: ______________________________

Instructor: __________________________

MATH 10560, Exam 3
April 23, 2013

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
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5.	(a)	(b)	(c)	(d)	(e)
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7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

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Multiple Choice	________
11.	________
12.	________
13.	________
Total	________

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Multiple Choice

1.(6 pts) Find the sum of the following series

$$\sum_{n=0}^{\infty} \frac{2^n + (-1)^n}{3^n}.$$

(a) $\frac{15}{4}$ (b) This series diverges. (c) $\frac{7}{4}$

(d) $\frac{5}{2}$ (e) $\frac{3}{2}$

2.(6 pts) Find the sum of the following series

$$\sum_{n=1}^{\infty} \left[\frac{5n}{n+3} - \frac{5(n+1)}{n+4}\right].$$

(a) $\frac{20}{4}$ (b) -5 (c) $\frac{5}{4}$

(d) $\frac{-15}{4}$ (e) This series diverges

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3.(6 pts) Consider the following series

$$(I)\quad \sum_{n=2}^{\infty} \frac{3n^3+2n+1}{2n^5+n^2} \qquad (II)\quad \sum_{n=2}^{\infty} \frac{n}{\ln(n)} \qquad (III)\quad \sum_{n=1}^{\infty} \frac{2^{n+1}}{3(n!)}$$

Which of the following statements is true?

(a) Only I and II converge

(b) Only I converges

(c) All three converge

(d) All three diverge

(e) Only I and III converge

4.(6 pts) Consider the following series

$$(I)\quad \sum_{n=1}^{\infty} \frac{\sin(n^2)}{n^2} \qquad (II)\quad \sum_{n=1}^{\infty} \frac{n^n}{(n^2+1)^n}$$

Which of the following statements is true?

(a) I is absolutely convergent by the comparison test

(b) I diverges because $\lim_{n\to\infty} \frac{\sin(n^2)}{n^2} \neq 0$

(c) I and II diverge by the alternating series test

(d) II diverges by the nth root test

(e) I converges by the alternating series test

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5.(6 pts) Which one of the following series converges conditionally?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{3^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^5}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{2^n}$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^4+1}$

6.(6 pts) Find a power series representation for the function

$$\frac{2x}{(1-x^2)^2}$$

in the interval $(-1, 1)$.
(Hint: Differentiate a well-known power series).

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n 2n x^{2n-1}$ (b) $\displaystyle\sum_{n=1}^{\infty} n x^{n-1}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{x^{2n+1}}{2n+1}$

(d) $\displaystyle\sum_{n=1}^{\infty} 2n x^{2n-1}$ (e) $\displaystyle\sum_{n=1}^{\infty} x^{2n}$

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7.(6 pts) Which of the series below has sum equal to $2e^{3/5}$?
Hint: A well-known power series may help.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n \cdot 3^n}{5^n(n!)}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{2 \cdot 3^n}{5^n(n!)}$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{6}{5(n!)}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{2 \cdot 3^{2n}}{5^{2n}[(2n)!]}$ (e) $\displaystyle\sum_{n=0}^{\infty} \frac{2 \cdot 3^{2n+1}}{5^{2n+1}[(2n+1)!]}$

8.(6 pts) The following is the fourth order Taylor polynomial of a function $f(x)$ at $a = 3$.

$$2 + \sqrt{3}(x-3) + 10(x-3)^2 + \pi(x-3)^3 + 3(x-3)^4$$

What is $f^{(2)}(3)$?

(a) 6π (b) 5 (c) 20 (d) $2\sqrt{3}$ (e) 3

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9.(6 pts) Compute the radius of convergence, R, of the following power series

$$\sum_{n=0}^{\infty} \frac{2x^n}{3^n n^2}$$

(a) $R = 3$ (b) $R = 2/3$ (c) $R = 1$

(d) $R = 1/3$ (e) $R = \infty$

10.(6 pts) Find the following limit using power series

$$\lim_{x \to 0} \frac{\sin(x^{10}) - x^{10}}{x^{30}}$$

(a) $-\frac{1}{3}$ (b) 1

(c) This limit does not exist. (d) 0

(e) $-\frac{1}{6}$

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Partial Credit

You must show your work on the partial credit problems to receive credit!

11. (13 pts.) Use the integral test to determine whether the series $\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$ converges or diverges.

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12. (14 pts.) Write out at least the first four terms for each series required below in addition to the general formula for the nth term; for example

$$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + \cdots.$$

(a) Write down the Taylor series expansion of $\cos(x)$ about 0.

(b) Use part (a) to find the Taylor series expansion of $\cos(x^2)$ about 0.

(c) Use part (b) to express $\int_0^{0.1} \cos(x^2)\, dx$ as a series.

(d) Use the alternating series estimation theorem to estimate $\int_0^{0.1} \cos(x^2)\, dx$ with a maximum error of 10^{-8}.

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13. (13 pts.) Find the radius of convergence and interval of convergence of the following series

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x-4)^n}{2^n (n+2)}.$$

Radius of Convergence : __________ **Interval of Convergence :** __________

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The following is the list of useful trigonometric formulas:

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}\big(\sin(x - y) + \sin(x + y)\big)$$

$$\sin x \sin y = \frac{1}{2}\big(\cos(x - y) - \cos(x + y)\big)$$

$$\cos x \cos y = \frac{1}{2}\big(\cos(x - y) + \cos(x + y)\big)$$

$$\int \sec\theta = \ln|\sec\theta + \tan\theta| + C$$

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